ELECTROPRODUCTION OF VECTOR MESONS AT SMALL x

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Abstract

Vector meson electroproduction is analyzed within the two-gluon (2G) model and the generalized parton distribution (GPD) approach at small x-Bjorken. We demonstrate that 2G and GPD models are not completely equivalent. At the same time, both models are in reasonable agreement with available experimental data on light vector meson electroproduction.

1 Vector meson production in 2G and GPD models

This report is devoted to investigation of vector meson electroproduction at small Bjorken x and large photon virtuality. In the low - x region the predominant contribution to the process is determined by the 2G exchange and the vector meson is produced via the photon-two-gluon fusion. At large Q^2 the cross section for the vector meson production is dominated by the $\gamma_L^* \to V_L$ amplitude which factorizes [1] into a hard meson photoproduction off gluons, and GPD. The amplitudes of $\gamma_\perp^* \to V_\perp$ and the $\gamma_\perp^* \to V_L$ transitions which are important in polarized observables are suppressed as a power of $1/Q$ and exhibit infrared singularities [2]. Similar properties of vector meson production amplitudes were found within the 2G model by several authors [3]. Calculation of these higher twist amplitudes requires a regularization scheme which depends on a model. The modified perturbative approach (MPA) [4] which includes the transverse quark motion gives possible ways of regularizing these end-point singularities. In this report, the MPA is used to study amplitudes of vector meson electroproduction for longitudinally and transversely polarized photons within the 2G and GPD models. Singularities in the amplitudes occurring in collinear approximation are regularized by the transverse quark momentum.

The leading twist term of the wave function gives a vanishing contribution to the amplitudes with a transversally polarized vector meson in the massless limit. To calculate these amplitudes, it is necessary to include in consideration the higher twist terms in the wave function. In this report, we use the k- dependent wave function [5]

$$\hat{\Psi}_V = g[(\not{V} + M_V)\not{E}_V + \frac{2}{M_V}\not{V}\not{E}_V\not{K} - \frac{2}{M_V}(\not{V} - M_V)(E_V \cdot K)]\phi_V(k_\perp^2, \tau). \qquad (1)$$

Here V is a momentum and M_V is a mass of a vector meson, E_V is its polarization, τ is a fraction of momentum V carried by the quark, and K is its transverse momentum: $K^2 = -k_\perp^2$. The first term in (1) represents the standard wave function of the vector meson. The leading twist contribution to the longitudinal vector meson polarization is determined by the $M_V\not{E}_V$ term in (1). The k- dependent terms of the wave function are essential for the amplitude with transversely polarized light mesons.

Let us consider vector meson production in MPA within the 2G model. The leading over s term of the $\gamma^\star \to V$ amplitude is mainly imaginary. The imaginary part of the amplitude can be written as an integral over τ and $k_\perp$ and has the form [6, 7]

$$T^V_{\lambda_V,\lambda_\gamma} = N \int d\tau \int dk_\perp^2 \frac{H^g(\xi,\xi,t)\,\phi_V(k_\perp^2,\tau)\,A_{\lambda_V,\lambda_\gamma}(\tau,k_\perp^2)}{(k_\perp^2+\bar{Q}^2)^3}, \tag{2}$$

where N is the normalization constant, $\bar{Q}^2 = \tau\bar{\tau}Q^2$, $\bar{\tau} = 1-\tau$. Positive proton helicities are omitted here for simplicity. In calculation of (2) the Feinman gauge is used and t-channel gluons are polarized longitudinally. The function $H^g(\xi,\xi,t)$ is connected to the gluon GPD at $x=\xi$ point [7], where skewness ξ is related to Bjorken-x by $\xi \simeq x/2$. The meson wave function ϕ_V is used in a simple Gaussian form [8]

$$\phi_V(k_\perp^2,\tau) \;=\; 8\pi^2\sqrt{2N_c}\,a_V^2\;\exp\left[-a_V^2\,\frac{\mathbf{k}_\perp^2}{\tau\bar{\tau}}\right]. \tag{3}$$

Transverse momentum integration of (3) leads to the asymptotic form of a meson distribution amplitude $\phi_V^{AS} = 6\tau\bar{\tau}$.

The hard amplitudes $A_{\lambda_V,\lambda_\gamma}$ in (2) are calculated perturbatively. The $\gamma_L^* \to V_L$ amplitude has the form [7]

$$A_{L,L} = 4\frac{s}{\sqrt{Q^2}}\left[\bar{Q}^2 + k_\perp^2(1-4\tau\bar{\tau})\right]\left(\bar{Q}^2+k_\perp^2\right). \tag{4}$$

For the amplitude with transversely polarized photons and vector mesons we find

$$A_{T,T} \sim \frac{2s}{M_V}\bar{Q}^2\left[k_\perp^2(1+4\tau\bar{\tau}) + 2M_V^2\tau\bar{\tau}\right]\,(E_\perp^\gamma E_\perp^V). \tag{5}$$

For the light meson production the resulting amplitude is proportional to $k_\perp^2$. The term proportional to M_V^2 appears in the amplitude for heavy mesons too.

The $\gamma_T^* \to V_L$ transition amplitude is determined by the function

$$A_{L,T} \sim \frac{2s}{M_V}\bar{Q}^2\left[2M_V^2\tau\bar{\tau} - k_\perp^2(1-2\tau)\right]\,\frac{(E_\perp^\gamma r_\perp)}{M_V}. \tag{6}$$

It can be found that if we omit the k^2 terms in the denominator of (2), the $T_{T,T}$ and $T_{L,T}$ amplitudes will have the end-point singularities at $\tau(\bar{\tau}) = 0$ [7]. All amplitudes in the 2G model are mainly imaginary. The real part of the amplitude can be obtained from the imaginary part using the derivative rule

$$\mathrm{Re}\,T \sim -\frac{\pi}{2}\frac{d}{d\ln x}\,\mathrm{Im}\,T\,. \tag{7}$$

The real parts of the amplitudes are small, about 30% with respect to its imaginary part.

The vector meson electroproduction can be studied within the GPD approach at large photon virtuality Q^2. At small Bjorken-x we shall consider as before the predominated gluon contribution. The $\gamma_L^* \to V_L$, $\gamma_T^* \to V_T$, $\gamma_T^* \to V_L$ amplitudes are calculated within the MPA. In the GPD model we consider the Sudakov suppression of large quark-antiquark separations. These effects provide additional suppression of contributions from

the end-point regions, in which one of the quarks entering into the meson wave function becomes soft and factorization breaks down. As previously, including the transverse quark momenta regularizes singularities and gives a possibility of calculating the transition amplitudes at large Q^2 which are important for polarized observables. The amplitudes $\gamma^*_\mu p \to V_{\mu'} p$ can be represented in the form [9]:

$$\begin{aligned} T_{\mu',\mu} &= \frac{e}{2} C_V \int_0^1 \frac{d\bar{x}}{(\overline{x}+\xi)(\overline{x}-\xi+i\hat{\varepsilon})} \\ &\times \left\{ \left[\mathcal{H}^{(g)}_{\mu'+,\mu+} + (-1)^{\mu'+\mu} \mathcal{H}^{(g)}_{-\mu'+,-\mu+} \right] H^g(\overline{x},\xi,t) \right\}, \end{aligned} \tag{8}$$

The flavor factor for ρ -meson production is $C_\rho = 1/\sqrt{2}$.

The hard scattering amplitudes $\mathcal{H}$ in (8) are written for the positive transverse gluon polarization and can be represented as a convolution of the hard part $A^{(g)}_{\mu',\mu}$, which is calculated perturbatively, and the wave function (3)

$$\mathcal{H}^{V(g)}_{\mu'+,\mu+} = \frac{2\pi\alpha_s(\mu_R) f_V}{N_c} \int_0^1 d\tau \int \frac{d^2\mathbf{k}_\perp}{16\pi^3} \phi_V(k^2_\perp,\tau)\, A^{(g)}_{\mu',\mu}(x,\xi,\mathbf{k}_\perp,Q^2)\,. \tag{9}$$

Here the scale μ_R is determined by the largest mass scale appearing in the hard scattering amplitude: $\mu_R = \max\{\tau Q,\, \bar{\tau} Q,\, ...\}$.

2 Amplitude structure and description of experiment

The GPD model leads to the following form of helicity amplitudes

$$T_{LL} \propto 1\,; \quad T^{V(g)}_{TT} \propto \frac{|\mathbf{k}_\perp|}{Q}\,; \quad T^{V(g)}_{TL} \propto \frac{\sqrt{-t}}{Q}\,. \tag{10}$$

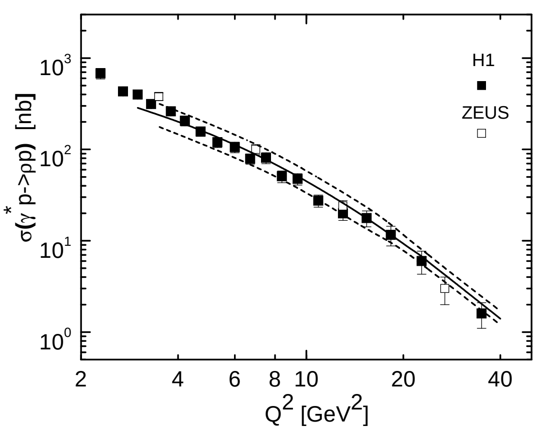
This behavior is similar to those obtained in the 2G model.

The 2G and GPD approaches are hoped to be equivalent at small x. Unfortunately, the amplitude structure in the models are not equivalent. As mentioned before, in the 2G model all amplitudes are mainly imaginary. In the GPD approach the integration over x occurs in (8)

$$\begin{aligned} T^{V(g)} &\sim \int_0^1 \frac{d\bar{x}\, H(\bar{x})}{(\overline{x}+\xi)(\overline{x}-\xi+i\hat{\varepsilon})} = I(\bar{x}<\xi) + I(\bar{x}>\xi) \\ &= \int_0^\xi \frac{d\bar{x}\, H(\bar{x})}{(\overline{x}+\xi)(\overline{x}-\xi+i\hat{\varepsilon})} + \int_\xi^1 \frac{d\bar{x}\, H(\bar{x})}{(\overline{x}+\xi)(\overline{x}-\xi+i\hat{\varepsilon})} \end{aligned} \tag{11}$$

For the nonflip T_{LL} and T_{TT} amplitudes we have no singularities in integrated functions $H(\bar{x})$, and both $I(\bar{x}<\xi)$ and $I(\bar{x}>\xi)$ contribute to the Re part of amplitude. These integrals are not small, have different signs and compensate each other mainly. As a result, the real part of the LL and TT amplitudes is quite small and is consistent with the one obtained from (7). In the case of the T_{LT} amplitude we have quite a different result. In this case, we find an additional coefficient $1/\sqrt{s\,u} \propto 1/\sqrt{\overline{x}^2-\xi^2}$ in the hard amplitude H in (11) which becomes imaginary in the $\overline{x}<\xi$ integration region. Consequently, the real

part of the T_{LT} amplitude is determined only by $I(\bar{x} > \xi)$ integral. This contribution is not small and we find that Re $T_{LT} >$ Im T_{LT} for this amplitude. Thus, properties of the T_{LT} amplitude in the 2G and GPD models are quite different. It is difficult to imagine that these amplitudes might be equivalent at small x.



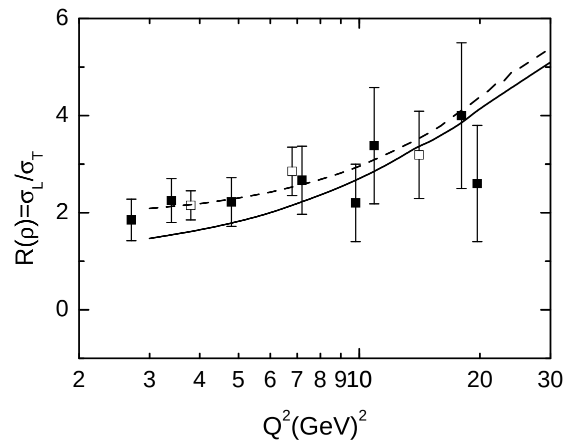


Figure 1: Left: The cross section for $\gamma^* p \to \rho^0 p$ vs. Q^2 for fixed values of $\langle W \rangle = 75\,\mathrm{GeV}$. Full line- GPG model results. Dashed lines show the μ_R sensitivity. Data are from [10, 11].

Figure 2: Right: Q^2 dependence R of ρ production at $\langle W \rangle = 75\,\mathrm{GeV}$. Full curve -2G model, dashed curve -GPD results. Data are from [10, 11].

Let us consider the description of experimental data in the 2G and GPD models. In both the cases we have the a_v parameter in the wave function which determines the average value of $\langle k_\perp^2 \rangle$ in hard subprocess. In the numerical evaluation of meson electroproduction a reasonable description of experimental data is obtained for $a_\rho = 0.8$ GeV^{-1} in the 2G model and for $a_\rho = 0.52$ GeV^{-1} in the GPD model. The parameter f_v is determined by the standard value and for ρ meson production we use $f_\rho = 0.216$ GeV. Estimations of the amplitudes are carried out using the $\Lambda_{QCD} = 0.22\,\mathrm{GeV}$. The cross section for $\gamma^* p \to \rho p$ production integrated over t is shown in Fig. 1 (full line). Good agreement with experiment is to be observed. The results for ϕ production can be found in [9]. It is important to analyse the dependence of cross section on the scale μ_R. The results for cross section for $\tilde{\mu}_R = \{\sqrt{2}\mu_R, \mu_R/\sqrt{2}\}$ are shown in Fig.1 by dashed lines. It can be seen that the $\tilde{\mu}_R$ sensitivity of the cross section is of the order of experimental errors.

Using the calculated amplitudes we can determine contributions to the cross section with longitudinal and transverse photon polarization and its ratio as

$$N_L = |T^V_{LL}|^2\,, \quad N_T = |T^V_{TT}|^2 + |T^V_{LT}|^2\,, \quad R = \frac{N_L}{N_T}. \tag{12}$$

Note that in (12) summation over proton helicities is assumed. We omit here the T_{TL} and T_{-TT} amplitudes which are small in the models. In terms these quantities the spin-density matrix elements (SDME) can be defined, e.g.

$$r^{04}_{00} = \frac{1}{N_T + \varepsilon N_L}\,(|T^V_{LT}|^2 \; + \varepsilon\, |T^V_{LL}|^2). \tag{13}$$

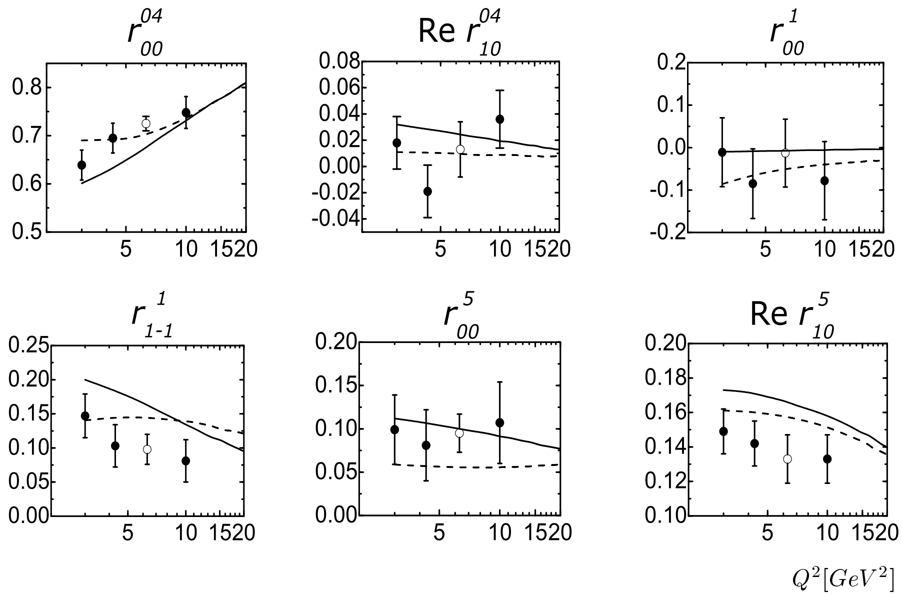


Figure 3: Q^2 dependence SDME of ρ production at $\langle -t\rangle = 0.15$ GeV^2 and $\langle W\rangle = 75$ GeV. Full curve -2G model, dashed curve -GPD results. Data are taken from [10, 11].

The model results for the ratio of cross section R are shown in Fig. 2. For both the models this ratio is growing with Q^2 and in consistent with experiment. In Fig.3, we show six essential SDME. In the approximation, when we put the T_{TL} and T_{-TT} amplitudes to be zero, the other SDME are connected with the matrix elements from Fig. 3 or equal to zero. The description of experimental data in both the models is reasonable.

We would like to note that from the data on $d\sigma/dt$ the diffraction peak slope $B \sim 6\,\mathrm{GeV}^{-2}$ at $Q^2 \sim 5\,\mathrm{GeV}^2$ can be determined [10, 11]. This value is connected with the diffraction peak slope of the T_{LL} amplitude because its contribution to the cross section is most essential. The diffraction peak slopes of the T_{TL} and T_{TT} amplitudes are not well defined. In calculation of spin observables we suppose that the diffraction peak slope $B_{LT} \sim B_{LL}$ and B_{TT} might be different. The slope $B_{TT} \sim B_{LL}$ in the 2G model and $B_{TT} \sim B_{LL}/3$ in the GPD model is used. Predictions of both the models are in agreement with the known t -dependence of experimental data at small momentum transfer [10]. The results found in [3] are very close to estimations obtained here within the 2G model (Fig.3).

3 Conclusion

Light vector meson electroproduction at small x was analyzed in this report within the 2G and GPD models. In both the models the amplitudes were calculated using MPA and the wave function (1) which consider the transverse quark momentum. By including the higher twist effects $k_\perp^2/Q^2$ in the denominators of $T^V_{\lambda_V,\lambda_\gamma}$ in (2) we regularize the end-point singularities in the amplitudes with transversally polarized photons. It was found that the 2G and GPD models, which are expected to be equivalent at small x, lead to similar results for the leading twist T_{LL} amplitude. At the same time, properties of the amplitudes suppressed as a power of $1/Q$ are different in the models. This was demonstrated here for the T_{LT} amplitude. Thus, the 2G and GPD models are not completely equivalent at small

x. It was shown that the diffraction peak slopes of the T_{TT} and T_{LT} amplitudes are not well defined. The knowledge of these slopes is essential in analyses of SDME. Information about B_{TT} and B_{LT} can be obtained from t -dependence of SDME.

At the same time, both approaches lead to an accurate description of the cross section for the light meson production. We found a reasonable results for SDME and R ratio in the 2G and GPD models. This means that at the present time we have two solutions for the scattering amplitudes which are in agreement with existing experimental data. Unfortunately, all data on spin observables have now large experimental errors. This does not permit one to determine which model is relevant to experiment. To clarify the situation, an additional theoretical study of the T_{TT} and T_{LT} amplitudes is needed. An experimental investigation to reduce errors in SDME is extremely important. Study of t dependence of SDME can give important information on either diffraction peak slopes in helicity amplitudes are of the same order of magnitude or different.

This work is supported in part by the Russian Foundation for Basic Research, Grant 03-02-16816 and by the Heisenberg-Landau program.

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